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PRESSURE GRADIENTS DUE TO FRICTION DURING THE FLOW OF EVAPORATING TWO-PHASE MIXTURES IN SMOOTH TUBES AND CHANNELS

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Abstract—The graphical procedure of Baroczy and equations for predicting local pressure gradients during the turbulent flow of two-phase mixtures in smooth tubes are transformed to enable their more convenient application to the case of evaporating flow. The resulting equations also prove to be convenient for use in predicting local pressure gradients. Limited comparison is made with data for flow in channels.

NOMENCLATURE

A, tube cross-section;
B, coefficient in equations (24) and (26);
C, coefficient in equation (11);
C_1, coefficient in Blasius' equation (14);
C_2, coefficient defined by equation (15);
D, defined by equation (28);
d, tube diameter;
G, mass velocity;
K, velocity ratio, equation (8);
l, length;
l_0, length over which evaporation occurs;
M, mass flowrate;
n, exponent in Blasius' equation (14);
Δp_G, pressure gradient if gas flows alone;
Δp_{GO}, pressure gradient due to friction if total mixture flows as gas;
Δp_L, pressure gradient due to friction if liquid flows alone;
Δp_{LO}, pressure gradient due to friction if total liquid flows as liquid;
Δp_{TP}, pressure gradient during two-phase flow;
q, dryness fraction;
q_0, dryness fraction at end of evaporating length;
Re, Reynolds number;
U_G, vapour velocity;
U_L, liquid velocity;
X, Lockhart–Martinelli parameter, equation (9);
α, ratio of gas to total cross-section;
Γ, physical property coefficient, equation (18);
λ, friction factor;
λ_{GO}, friction factor when all of mixture flows as gas;
λ_{LO}, friction factor when all of mixture flows as liquid;
λ_M, friction factor when mixture flows;
μ_G, absolute viscosity of gas;
μ_L, absolute viscosity of liquid;
ρ_G, density of gas;
ρ_L, density of liquid;
ϕ_{LO}^2, the Lockhart–Martinelli two-phase multiplier, $\Delta p_{TP}/\Delta p_{LO}$;
ψ_1, group given by equation (29).

1. INTRODUCTION

THE PREDICTION of pressure gradients during the flow of two-phase mixtures is an essential step in the design of a great variety of industrial plant in the power and process industries. Despite the considerable progress made in recent years [1–3] in this field, a considerable need exists for convenient, rapid, and accurate estimation procedures.

This is particularly the case in relation to evaporating flow where most existing prediction procedures require the pressure drop due to friction to be evaluated by arithmetic integration. In this paper equations for the prediction of local friction pressure gradients during the turbulent flow of two-phase mixtures are developed in a form which permits their ready integration to give the overall pressure drop during evaporation.

2. AN ELEMENTARY MODEL

One elementary model [4] expresses the frictional resistance during two-phase flow through pipes in terms of the "true dynamic head" of the mixture as follows:

$$\Delta p_{TP} = \frac{\lambda_M}{2d}\{\alpha U_G^2 \rho_G + (1-\alpha) U_L^2 \rho_L\}. \quad (1)$$

It can be seen that at the all-liquid and all-gas (or all-vapour) conditions this reduces to the normal expression for single-phase flow.

The continuity equations for the vapour and liquid are respectively

$$(1-q) M = (1-\alpha) A U_L \rho_L \quad (2)$$

$$qM = \alpha A U_G \rho_G. \quad (3)$$

If the components flowed alone their friction pressure gradients would be, where the friction factor is independent of Reynolds number,

$$\Delta p_L = \frac{\lambda (1-q)^2 M^2}{2d\rho_L A^2}; \quad (4)$$

$$\Delta p_G = \frac{\lambda q^2 M^2}{2d\rho_G A^2}. \quad (5)$$

Combining equations (1)–(5), assuming $\lambda_M = \lambda$, gives

$$\frac{\Delta p_{TP}}{\Delta p_{LO}} = \frac{q^2 \rho_L}{\alpha \rho_G} + \frac{(1-q)^2}{1-\alpha}. \quad (6)$$

From equations (2) and (3)

$$\frac{1}{\alpha} = 1 + K \frac{1-q}{q}\frac{\rho_G}{\rho_L} \quad (7)$$

where the velocity ratio K is

$$K = \frac{U_G}{U_L}. \quad (8)$$

3. LOCKHART–MARTINELLI PARAMETER

Lockhart and Martinelli [5] correlated a considerable amount of data by plotting $\Delta p_{TP}/\Delta p_L$ to a base of the parameter

$$X^2 = \frac{\Delta p_L}{\Delta p_G}. \quad (9)$$

From equations (4), (5) and (9), where λ is independent of Reynolds number,

$$X^2 = \left(\frac{1-q}{q}\right)^2 \frac{\rho_G}{\rho_L}. \quad (10)$$

Combining equations (6), (7) and (10) gives

$$\frac{\Delta p_{TP}}{\Delta p_L} = 1 + \frac{C}{X} + \frac{1}{X^2}, \quad (11)$$

where

$$C = \frac{1}{K}\sqrt{\left(\frac{\rho_L}{\rho_G}\right)} + K\sqrt{\left(\frac{\rho_G}{\rho_L}\right)}. \quad (12)$$

Equations (11) and (12) have been developed previously [6, 7] by the writer for situations other than considered here.

While the model is too crude to warrant much expectation of close agreement with experiment, nevertheless the empirical curve of Lockhart and Martinelli closely approximates to equation (11) with $C = 21$, as can be seen in Fig. 1 taken from a paper by Chisholm and Laird [8]. The data were obtained with smooth tubes in which case

$$X^2 = \left(\frac{1-q}{q}\right)^{2-n} \frac{\rho_G}{\rho_L}\left(\frac{\mu_L}{\mu_G}\right)^n, \quad (13)$$

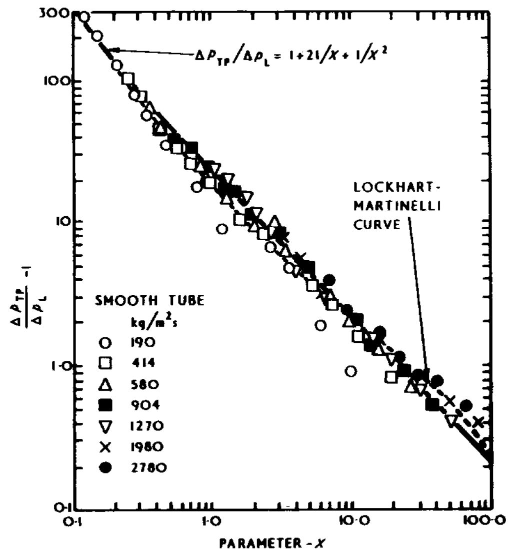


FIG. 1. $\Delta p_{TP}/\Delta p_L$ to a base of Lockhart–Martinelli parameter X: air/water mixtures in a 27 mm bore horizontal tube at atmospheric pressure.

where n is the exponent in the Blasius relation for friction factor

$$\lambda = \frac{C_1}{Re^n}. \tag{14}$$

Equation (13) reduces to equation (10) when $n = 0$; it has not yet proved possible to derive equation (11) except where $n = 0$.

Chisholm [9] and Sutherland [10] present values of C and C_2 defined by

$$C_2 = C\Big/\left\{\sqrt{\left(\frac{\rho_L}{\rho_G}\right)} + \sqrt{\left(\frac{\rho_G}{\rho_L}\right)}\right\} \tag{15}$$

for a range of conditions. Comparison of equations (12) and (15) indicates that for homogeneous flow ($K = 1$) the coefficient C_2 is unity. For flow of steam/water mixtures in tubes of commercial roughness Thom [11] recommended

$$C = 1{\cdot}1\left\{\sqrt{\left(\frac{\rho_L}{\rho_G}\right)} + \sqrt{\left(\frac{\rho_G}{\rho_L}\right)}\right\} - 0{\cdot}2 \tag{16}$$

and later [12]

$$C = 1 + \frac{q/\rho_G}{(q/\rho_G) + (1-q)/\rho_L} - \alpha. \tag{17}$$

Use of equations (16) and (17) will give predicted values slightly in excess of homogeneous theory.

4. PHYSICAL PROPERTY COEFFICIENT

Baroczy [13] introduced a "physical property index"

$$\frac{\rho_G}{\rho_L}\left(\frac{\mu_L}{\mu_G}\right)^{0\cdot 2}.$$

There are a number of advantages [14] however in defining a physical property coefficient

$$\Gamma = \left(\frac{\Delta p_{GO}}{\Delta p_{LO}}\right)^{0\cdot 5}. \tag{18}$$

The pressure gradients are those if the whole mixture flows as vapour or liquid,

$$\Delta p_{LO} = \frac{\lambda_{LO} G^2}{2d\rho_L} \tag{19}$$

and

$$\Delta p_{GO} = \frac{\lambda_{GO} G^2}{2d\rho_G} \tag{20}$$

where G is the mass velocity of the mixture.

For turbulent flow in smooth tubes

$$\Gamma = \left(\frac{\rho_L}{\rho_G}\right)^{0\cdot 5}\left(\frac{\mu_G}{\mu_L}\right)^{n/2} \tag{21}$$

and in rough tubes ($n = 0$) where

$$\Gamma = \left(\frac{\rho_L}{\rho_G}\right)^{0\cdot 5}. \tag{22}$$

From equations (13) and (21)

$$X = \left(\frac{1-q}{q}\right)^{(2-n)/2}\Big/\Gamma \tag{23}$$

5. PROPOSED EQUATIONS

Equation (11) is in rather an unsatisfactory form for use with evaporating flows as Δp_L, in that case, varies along the flow path. More convenient forms of this equation for use with evaporating flow are now presented.

The writer has already shown [14] that for $n = 0$ from equations (9)–(12), (15) and (22)

$$\frac{\Delta p_{TP}}{\Delta p_{LO}} = 1 + (\Gamma^2 - 1)\{Bq(1 - q) + q^2\} \quad (24)$$

where

$$B = \frac{C\Gamma - 2}{\Gamma^2 - 1} = \frac{C_2(\Gamma^2 + 1) - 2}{\Gamma^2 - 1}. \quad (25)$$

From equation (25) it is apparent that, where $\Gamma^2 \gg 1$, $B \doteqdot C_2$. From equations (12) and (15) it follows that B is unity for homogeneous flow ($K = 1$).

It should be noted that, regardless of the value of B, equation (28) satisfies the following important boundary conditions:

$$\begin{aligned} q &= 0 \quad \Delta p_{TP} = \Delta p_{LO} \\ q &= 1 \quad \Delta p_{TP} = \Delta p_{GO} \\ \Gamma^2 &= 1 \quad \Delta p_{TP} = \Delta p_{LO} \\ & \qquad\qquad = \Delta p_{GO}. \end{aligned}$$

Equations (26) and (27), of course, reduce to equations (24) and (25) when $n = 0$. As C is independent of dryness fraction q, so also is B.

The precise transformation of equation (11) contains a further term D on the right-hand side of equation (26). The term D is

$$D = (1 - q)^{2-n} + (2^{2-n} - 2)\, q^{(2-n)/2} \times (1 - q)^{(2-n)/2} + q^{2-n} - 1. \quad (28)$$

With $n = 0{\cdot}25$, D has a maximum value [15] of about 0·025 as shown in Table 1 and can therefore be neglected in engineering calculations.

6. GRAPHICAL PRESENTATION OF TWO-PHASE FLOW DATA

The form of equation (26) suggests that a useful method of presenting data for two-phase flow is by plotting the group

$$\frac{(\Delta p_{TP}/\Delta p_{LO}) - 1}{\Gamma^2 - 1} = \psi_1 \quad (29)$$

Table 1. The term D *as a function of the dryness fraction* q (n = 0·25)

Dryness fraction q	0·000	0·010	0·050	0·200	0·500	0·800	1·000
D	0·000	0·023	0·015	0·011	0·000	0·011	0·000

Earlier [14] the writer had been unsuccessful in transforming equation (11) to the general form of equation (24) in the case of smooth tubes. The required transformation of equation (11) has now been obtained and is approximately

$$\frac{\Delta p_{TP}}{\Delta p_{LO}} = 1 + (\Gamma^2 - 1)\{Bq^{(2-n)/2}(1 - q)^{(2-n)/2} + q^{2-n}\}, \quad (26)$$

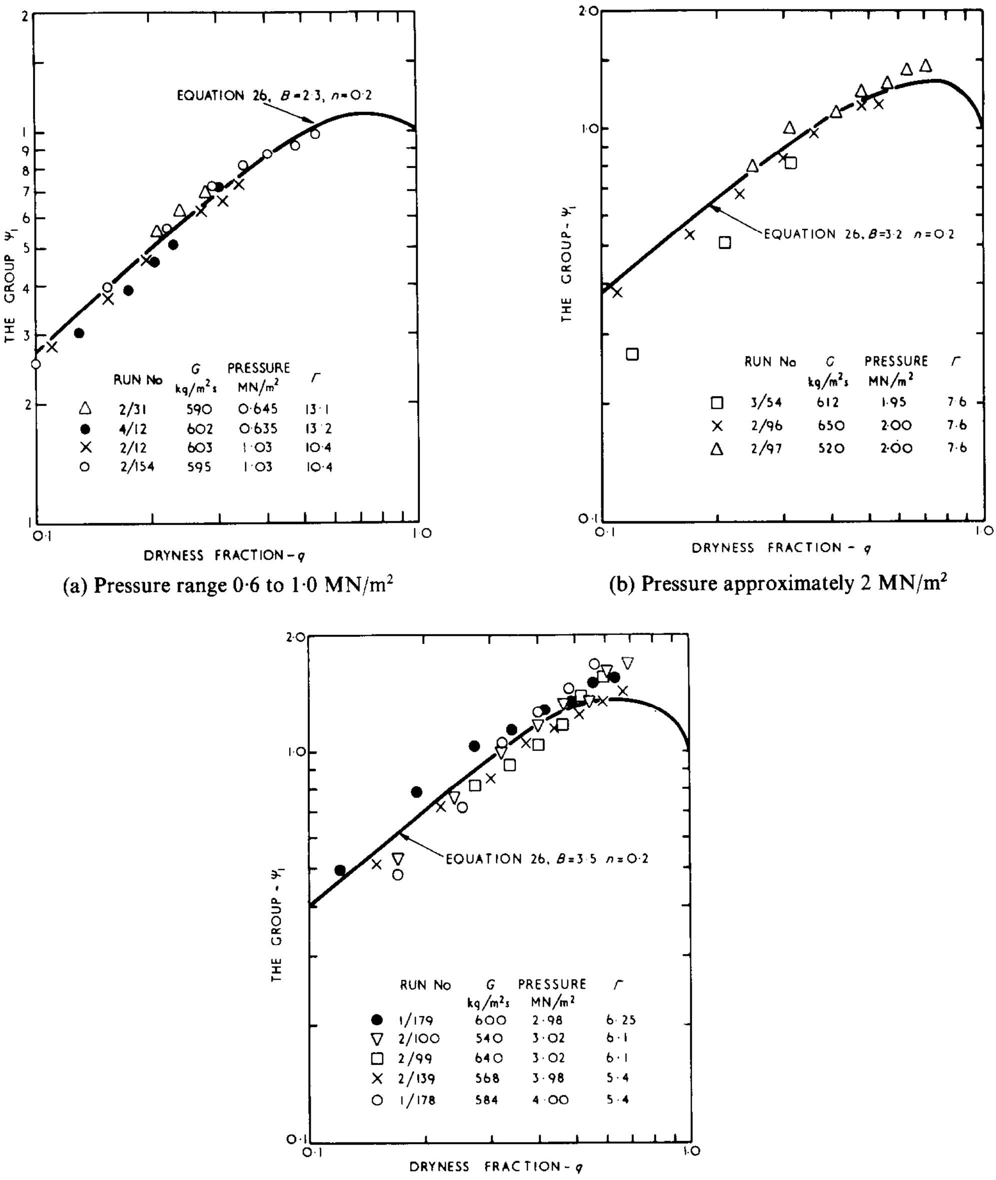
where

$$B = \frac{C\Gamma - 2^{2-n} + 2}{\Gamma^2 - 1}. \quad (27)$$

to a base of q; for convenience the group has been designated ψ_1. It has been shown in [14] that the family of curves in the well-known correlation of Chenoweth and Martin [16] reduces to a single curve in this type of plot. For homogeneous flow in rough tubes as $B = 1$ equation (26) can be expressed

$$\frac{(\Delta p_{TP}/\Delta p_{LO}) - 1}{\Gamma^2 - 1} = q. \quad (30)$$

Thus the proposed graphical plot is identical to



(a) Pressure range 0·6 to 1·0 MN/m²

(b) Pressure approximately 2 MN/m²

(c) Pressure range 3–4 MN/m²

FIG. 2. The group ψ_1 to a base of q: steam/water mixtures in vertical tubes with $G \doteqdot 600$ kg/m²s.

that used in a paper by James [17]; when correlating data for steam/water flow through nozzles, he plotted the dryness fraction that made experiment agree with homogeneous theory against the actual dryness fraction. This is in fact plotting ψ_1 against q.

Figure 2 presents, in the manner discussed above, some of the data of Becker, Hernborg and Bode [18] for steam/water flow in vertical tubes. As B is a function of both the mass velocity and the physical properties of the mixture, the data in the three parts of Fig. 2 are for a mass velocity of about 600 kg/m^{2}s, and each corresponds to a particular range of pressure. Figure 2 also shows the curves of ψ_1 obtained from equation (26) with the arbitrary values of B given on the figures. Methods of obtained the values of B will now be examined.

7. VALUES OF THE COEFFICIENT *B*

Using the Baroczy [13] correlation as a basis, Chisholm and Sutherland [10] obtained a graphical plot of C (equation (11)) as a function of Γ and G. This has been transformed to the plot of B as a function of Γ and G shown in Fig. 3; the points associated with the curves corresponding to mass velocities of 339 and 4029 kg/m^{2}s were evaluated from the figures in Table 4·2 of [10]. Figure 4 is a cross-plot of Fig. 3 and presents B as a function of G.

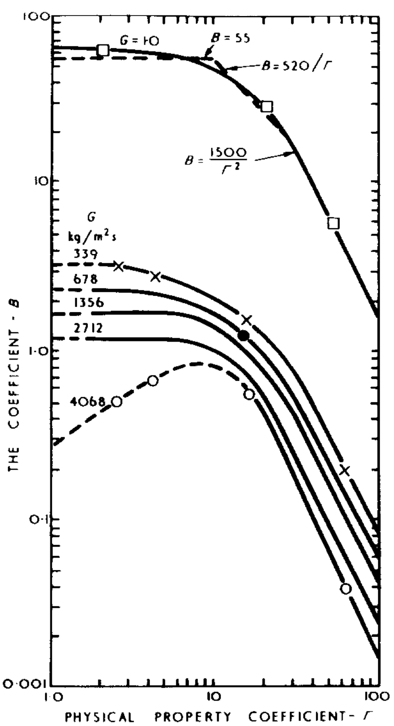


FIG. 3. The coefficient B to a base of Γ: based on Baroczy [13] correlation.

From Fig. 4 it can be seen that B tends to vary as $1/G^{\frac{1}{2}}$. The lines of constant Γ have been extrapolated on this basis to give values of B at $G = 1$; these values at $G = 1$ are also plotted in Fig. 3. From this curve, values of B at other mass velocities can be obtained by dividing by $G^{\frac{1}{2}}$. The error in this approach can be assessed from Fig. 4; the error is the difference between

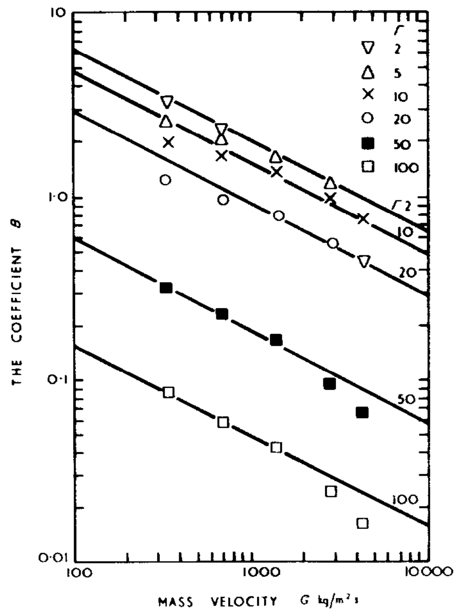


FIG. 4. The coefficient B to a base of mass velocity of mixture.

the lines and points at corresponding values of Γ. The largest error occurs at $G = 4068$ kg/m^{2}s and low Γ values; at $\Gamma = 2$ the predicted value will be almost twice the value of B obtained directly from the curve at this mass velocity in Fig. 3, but elsewhere the agreement is considerably more satisfactory.

The curve of B for $G = 1$ can be approximated by three straight lines as indicated in Fig. 3. This then leads to the following formulas for B which will approximate the graphs of Baroczy.

For

$$0 < 9{\cdot}5 \qquad B = 55/G^{\frac{1}{2}} \quad (31)$$

$$9{\cdot}5 < \Gamma < 28 \quad B = 520/(\Gamma G^{\frac{1}{2}}) \quad (32)$$

$$28 < \Gamma \qquad B = 15000/(\Gamma^2 G^{\frac{1}{2}}). \quad (33)$$

In these equations the mass velocity must have the units kg/m²s.

It should be noted that:

(a) for $G > 1900$ kg/m²s and $\Gamma < 9{\cdot}5$, equation (31) gives values smaller than from the writer's correlation [9], which is transformed into terms of B in Appendix I; and

(b) Chisholm and Laird's approximation to the Lockhart–Martinelli curve (equation (11), $C = 21$) corresponds, as shown in Appendix I, to $B \doteqdot 21/\Gamma$. The Lockhart–Martinelli correlation was developed within the region $9{\cdot}5 < \Gamma < 28$; combining equations (32) and (43) (with $C = 21$) indicates that the Baroczy procedure will give values smaller than the Lockhart–Martinelli procedure when $G > 600$ kg/m²s.

The writer therefore recommends that B should be evaluated as shown in Table 2. This is a compromise between the correlations of Baroczy, Lockhart–Martinelli, and Chisholm such that the greatest estimate of pressure gradient will be obtained; in engineering design this is normally to be preferred. It is of interest at this point to compare these recommendations with those of Sher and Green [19] for steam/water flow in vertical channels at a pressure of 13·8 MN/m². They presented ϕ_{LO}^2 as a function of the mass velocity of the mixture and the dryness fraction as shown in Table 3; the writer has added values of B obtained using the tabulated values of ϕ_{LO}^2 and equation (26) taking $n = 0{\cdot}2$. It can be seen that B, as anticipated, is essentially independent of dryness fraction but a strong function of the mixture mass velocity. Values are shown in the table only in the regions where data existed.

Table 2. Values of B for smooth tubes

Γ	G (kg/m²s)	B
	$\leqslant 500$	4·8
$\leqslant 9{\cdot}5$	$500 < G < 1900$	$2400/G$
	$\geqslant 1900$	$55/G^{0{\cdot}5}$
$9{\cdot}5 < \Gamma < 28$	$\leqslant 600$	$520/(\Gamma G^{0{\cdot}5})$
	> 600	$21/\Gamma$*
$\geqslant 28$		$\dfrac{15000}{\Gamma^2 G^{0{\cdot}5}}$

* This B corresponds to Lockhart–Martinelli curve (see Appendix I).

Table 3. Values of $\frac{\Delta p_{TP}}{\Delta p_{LO}}$ and B for steam/water mixtures at 14 MN/m² (Sher and Green [19])

Mixture mass velocity G (kg/m²s)	950		1080		1356		2030		2710		6770	
Dryness fraction q	$\frac{\Delta p_{TP}}{\Delta p_{LO}}$	B	$\frac{\Delta p_{TP}}{\Delta p_{LO}}$	B	$\frac{\Delta p_{TP}}{\Delta p_{LO}}$	B	$\frac{\Delta p_{TP}}{\Delta p_{LO}}$	B	$\frac{\Delta p_{TP}}{\Delta p_{LO}}$	B	$\frac{\Delta p_{TP}}{\Delta p_{LO}}$	B
0·00	1·00		1·00		1·00		1·00		1·00		1·00	
0·01	1·23	3·14	1·20	2·72	1·16	2·18	1·10	1·34	1·07	0·95	1·05	0·685
0·03	1·65	3·26	1·55	2·76	1·46	2·30	1·31	1·54	1·24	1·18	1·13	0·615
0·07	2·37	3·30	2·14	2·72	1·92	2·18	1·59	1·37	1·47	1·07		
0·10	2·87	3·35	2·56	2·76	2·23	2·15	1·78	1·31	1·61	1·00		
0·20	4·31	3·37	3·80	2·80	3·11	2·05	2·30	1·15				
0·30	5·59	3·51	4·83	2·86	3·84	2·00						
0·40	6·85	3·84	5·80	3·00	4·49	2·01						

It is relevant that the use of a constant value of B gives closer agreement with the data than the empirical curve of Sher and Green, as can be shown with reference to Fig. 5, which compares their empirical curve with their data at a mass velocity of 950 kg/m^{2}s. At a dryness

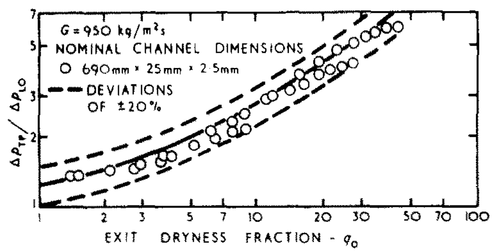


FIG. 5. $\frac{\Delta p_{TP}}{\Delta p_{LO}}$ to a base of q: steam/water mixtures in a vertical channel at a pressure of 13·8 MN/m^2 (Sher and Green [19]).

fraction of 0·4 Sher and Green's value for ϕ_{LO}^2 is 6·85, whereas taking $B = 3{\cdot}3$ gives $\phi_{LO}^2 = 6{\cdot}2$, which is in closer agreement with experiment; for dryness fractions below 0·3 the use of $B = 3{\cdot}3$ gives ϕ_{LO}^2 values indiscernible from Sher and Green's empirical curve.

Figure 6 compares the various methods of estimating B at a value of Γ corresponding to steam/water mixtures at a pressure of 13·8

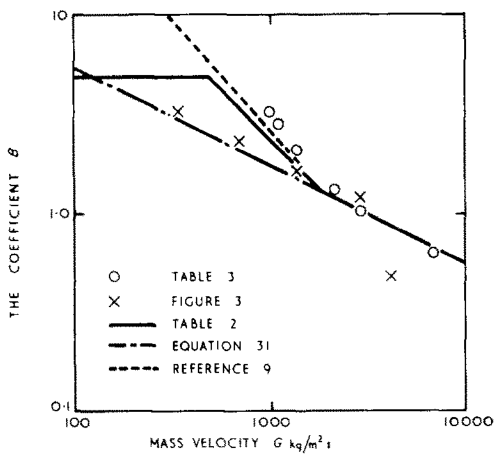


FIG. 6. Comparison of values of B from different sources: Γ corresponding to steam/water mixtures at 13·8 MN/m^2 ($\Gamma = 2{\cdot}39$).

MN/m^2 ($\Gamma = 2{\cdot}39$). The values corresponding to Sher and Green's correlation are slightly in excess of the values obtained by the recommendations of Table 2 at lower values of the mass velocity but are in excellent agreement at higher values; arithmetic mean values of B from Table 3 have been used.

Figure 7 compares the values of B corresponding to the curves in Fig. 2 with Baroczy's curve from Fig. 3 and with the equations from

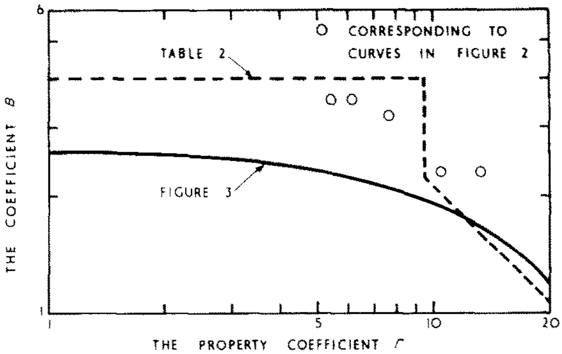


FIG. 7. The coefficient B to a base of Γ for $G = 600$ kg/m^{2}s.

Table 2, all corresponding to $G = 600$ kg/m^{2}s. One unsatisfactory aspect of the proposed procedures is the discontinuity at $\Gamma = 9{\cdot}5$. Further work is required to overcome this and other shortcomings of the present generalized approach. One step which should improve the accuracy of correlation is the development of procedures which acknowledge that the frictional pressure gradient is influenced by the inclination of the tube. The correlation of Baroczy, where $\Gamma < 9{\cdot}5$, has been developed using vertical tube data, whereas for $\Gamma > 9{\cdot}5$ the data were obtained on horizontal tubes.

8. PRESSURE DROP DUE TO FRICTION OVER EVAPORATING LENGTH

Where the change in pressure along a tube is sufficiently small in relation to the absolute pressure that Γ can be assumed constant, it is possible in certain cases to integrate equation (26) to give the contribution to the overall pressure drop due to friction. For the case where the dryness fraction varies linearly along the length

$$\frac{q}{q_0} = \frac{l}{l_0}. \qquad (34)$$

It follows from equation (26) that the average two-phase multiplier is

$$\frac{1}{\Delta p_{\mathrm{LO}} l_0} \int_0^{l_0} \Delta p_{\mathrm{TP}}\, \mathrm{d}l = 1 + (\Gamma^2 - 1)\left\{\frac{B}{q_0}\int_0^{q_0} q^{(2n-n)/2}(1-q)^{(2-n)/2}\,\mathrm{d}q + \frac{q_0^{2-n}}{3-n}\right\}. \qquad (35)$$

The expression

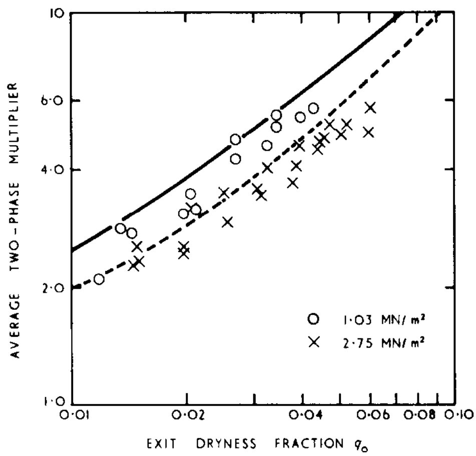
$$\frac{1}{q_0}\int_0^{q_0} q^{(2-n)/2}(1-q)^{(2-n)/2}\,\mathrm{d}q$$

can be evaluated using a series expansion as shown in Appendix II. Values for this expression are given in Table 4.

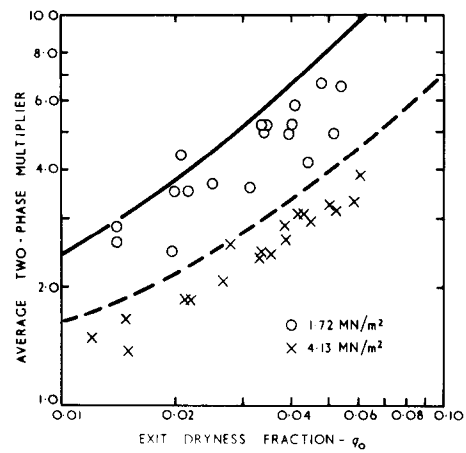
Table 4. Values of $\frac{1}{q_0}\int_0^{q_0} q^{(2-n)/2}(1-q)^{(2-n)/2}\,dq$

q_0 \ n	0·25	0·20	0·10
0·01	0·009 43	0·008 29	0·006 42
0·02	0·017 20	0·015 38	0·012 32
0·03	0·024 38	0·022 02	0·017 99
0·04	0·031 17	0·028 36	0·023 49
0·05	0·037 67	0·034 46	0·028 85
0·06	0·043 93	0·040 36	0·034 08
0·07	0·049 97	0·046 08	0·039 19
0·08	0·055 82	0·051 64	0·044 21
0·09	0·061 51	0·057 06	0·049 12
0·1	0·067 04	0·062 34	0·053 92
0·2	0·115 40	0·108 95	0·097 14
0·3	0·153 66	0·146 21	0·132 42
0·4	0·183 48	0·175 40	0·160 36
0·5	0·205 62	0·197 11	0·181 22
0·6	0·220 37	0·211 59	0·195 13
0·7	0·227 88	0·218 93	0·202 13
0·8	0·228 17	0·219 15	0·202 23
0·9	0·221 01	0·212 09	0·195 36
1·0	0·205 62	0·197 11	0·181 22

Figure 8 compares equation (35), using B values evaluated from Table 2, with the experimental data from [20] for the flow of steam/water mixtures in vertical channels. As in the



(a) Pressures of 1·03 and 2·75 MN/m²



(b) Pressures of 1·72 and 4·13 MN/m²

FIG. 8. The average two-phase multiplier to a base of exit dryness fraction q_0 at a mass velocity of 815 kg/m²s for steam/water mixtures in vertical channels.

case of the Becker data the recommended procedure tends to slightly over-estimate the value of the two-phase multiplier. The least satisfactory agreement is obtained at 1·72 MN/m² ($\Gamma = 7{\cdot}85$); this is associated with the discontinuities in the recommended procedures at $\Gamma = 9{\cdot}5$.

It is important to note that, while the procedures developed for smooth tubes give satisfactory agreement with the data for channels examined in this paper, these procedures considerably underestimate the data of Petrick [21] water flow in horizontal tubes where the mass velocity is below 700 kg/m^{2}s.

9. CONCLUSIONS

It has been shown that the equation for predicting pressure gradients during two-phase flow,

$$\frac{\Delta p_{TP}}{\Delta p_{L}} = 1 + \frac{C}{X} + \frac{1}{X^2}, \qquad (11)$$

can be transformed with sufficient accuracy for engineering purposes to

$$\frac{\Delta p_{TP}}{\Delta p_{LO}} = 1 + (\Gamma^2 - 1)\{Bq^{(2-n)/2}(1-q)^{(2-n)/2} + q^{2-n}\} \qquad (26)$$

where B is defined in terms of C in equation (27).

A method of graphical presentation of two-phase flow data suggested by the form of the latter equation has been discussed; for rough tubes this is essentially the procedure used by James [17].

The values of B corresponding to Baroczy's correlation are given in Fig. 3 and approximated by equations (31)–(33). There is evidence that the Baroczy correlation may underestimate the prediction of friction in certain situations, and for this reason the values of B in Table 2 are recommended.

Where the dryness fraction varies linearly along the tube, the pressure drop due to friction can be evaluated using equation (35).

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APPENDIX I

Approximate Equations for B

For the flow of steam/water mixtures in tubes at pressures above 3 MN/m^2 Chisholm [9] recommended the following equations for the evaluation of C with smooth tubes with mixture mass velocities less than 2000 kg/m^{2}s.

$$C = \frac{2000}{G}\left\{\left(\frac{\rho_L}{\rho_G}\right)^{0\cdot5} + \left(\frac{\rho_G}{\rho_L}\right)^{0\cdot5}\right\}. \quad (36)$$

A more complex equation was also given which ensured that C approached the correct value at the critical point. Equation (36) was primarily based on the data in [22]; the pressure range was from 4 to 7 MN/m^2.

Combining equations (27) and (36) gives, taking $n = 0{\cdot}2$,

$$B = \left[\frac{2000}{G}\left\{\left(\frac{\rho_L}{\rho_G}\right)^{0\cdot5} + \left(\frac{\rho_G}{\rho_L}\right)^{0\cdot5}\right\}\Gamma - 1{\cdot}5\right]\Big/(\Gamma^2 - 1). \quad (37)$$

At the upper limit of pressure in the Berkowitz data $\Gamma^2 = 15$, hence equation (37) can be approximated to

$$B = \frac{2000}{G}\left\{\left(\frac{\rho_L}{\rho_G}\right)^{0\cdot5} + \left(\frac{\rho_G}{\rho_L}\right)^{0\cdot5}\right\}\frac{1}{\Gamma} - \frac{1{\cdot}5}{\Gamma^2}, \quad (38)$$

and, in this pressure range, less than 5 per cent error is introduced by making the further approximation

$$B \doteqdot \frac{2000}{G}\left(\frac{\rho_L}{\rho_G}\right)^{0\cdot5}\frac{1}{\Gamma}. \quad (39)$$

Hence combining equation (39) with equation (21), and taking $n = 0{\cdot}2$

$$B \doteqdot \frac{2000}{G}\left(\frac{\mu_L}{\mu_G}\right)^{0\cdot1}. \quad (40)$$

Over the range of conditions for which equation (36) was derived, $(\mu_L/\mu_G)^{0\cdot1}$ varied between 1·174 and 1·214; this equation can therefore be finally approximated to

$$B \doteqdot \frac{2400}{G}. \quad (41)$$

It was recommended previously that, where $2000/G > 4$, the value of B should be made equal to 4. Hence if $B > 4{\cdot}8$ it should be taken as 4·8.

Equations (31) and (41) give identical values of B at 1900, so for convenience the range of applicability of equation (41) will be taken as $G > 1900$, rather than $G > 2000$ as in [9].

If the same approximations are made as in deriving equation (41), the relationship between B and C is

$$B = C/\Gamma. \quad (42)$$

APPENDIX II

Evaluation of $\int_0^{q_0} q^{(2-n)/2}(1 - q)^{(2-n)/2}\,dq$)

The integral can be evaluated using a series expansion as follows

$$\int_0^{q_0} q^{(2-n)/2}(1-q)^{(2-n)/2}\,dq = \int_0^{q_0} q^m(1-q)^m\,dq$$
$$= \frac{q_0^{m+1}}{m+1} - \frac{mq_0^{m+2}}{m+2} + \frac{m(m-1)}{2!(m+3)}q_0^{m+3}$$
$$- \frac{m(m-1)(m-2)}{3!(m+4)}q_0^{m+4} + \ldots \quad (43)$$

For values of q_0 up to 0·5 the use of four terms in the series gives an accuracy of 0·02 per cent with $n = 0{\cdot}25$. For q_0 between 0·5 and 1 a similar accuracy is obtained using the equation

$$\int_0^{q_0} q^m(1-q)^m\,dq = 0{\cdot}2056 - \int_0^{1-q_0} q^m(1-q)^m\,dq. \quad (44)$$

GRADIENTS DE PRESSION DUS AU FROTTEMENT LORS DE L'ECOULEMENT DE MELANGES BIPHASIQUES EN EVAPORATION DANS DES TUBES ET DES CANAUX LISSES

Résumé—La méthode graphique de Baroczy et les équations pour évaluer les gradients locaux de pression pour un écoulement turbulent d'un mélange biphasique dans des tubes lisses ont été transformées afin de permettre leur application convenable au cas de l'écoulement avec évaporation. Les équations qui résultent sont utilisables pour estimer les gradients locaux de pression. On a fait une comparaison limitée avec des résultats expérimentaux sur l'écoulement dans des canaux.

DRUCKGRADIENTEN INFOLGE VON REIBUNG BEI ZWEI-PHASEN-STRÖMUNGEN MIT VERDAMPFUNG IN GLATTEN ROHREN UND KANÄLEN

Zusammenfassung—Das graphische Verfahren von Baroczy und die Gleichungen zur Bestimmung lokaler Druckgradienten bei turbulenter Strömung von Zwei-Phasen-Gemischen in glatten Rohren wurden transformiert, um sie auf Verdampfungsströmungen anzuwenden.

Die resultierenden Gleichungen sind auch zur Bestimmung lokaler Druckgradienten geeignet.

Ein beschränkter Vergleich mit den Daten für Kanalströmungen wurde durchgeführt.

ГРАДИЕНТЫ ДАВЛЕНИЯ В РЕЗУЛЬТАТЕ ТРЕНИЯ ПРИ ТЕЧЕНИИ ИСПАРЯЮЩИХСЯ ДВУХФАЗНЫХ СМЕСЕЙ В ГЛАДКИХ ТРУБАХ И КАНАЛАХ

Аннотация—Графический метод Барочи и уравнения для расчета локальных градиентов давления при турбулентном течении двухфазных смесей в гладких трубах специально преобразованы для применения к расчету течений при испарении. Показано, что полученные уравнения применимы также для расчета локальных градиентов давления.

Проведено сравнение некоторых данных для течения в каналах.